Finding Plank's Constant Using the Photoelectric Effect

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To explore the photo-electric effect the value of Plank's constant was found along with the work function (Φ) of a photo-cathode in attempt to identify the metal. The value of Plank's constant was accurately determined to within 5% of the currently accepted value; however the work function of the photo-cathode was unrealistically low.

I. INTRODUCTION

A. Purpose

The purpose of this lab was to experimentally determine the value for Plank's constant. A secondary objective was to determine the element of a photo-cathode based on its work function. Determining an accurate value of Plank's constant is crucial for much of quantum physics because of its presence throughout the field. Identifying an unknown metal based on its work function may also be useful in scenarios where access to the material is limited.

B. Theory and Background

Einstein's explanation of the photoelectric effect further solidified the claims that light displays corpuscule properties. One of the first equations to use Plank's constant was the Plank-Einstein relation used to describe the energy of these particles of light, or photons.

$$E = hf \qquad \text{(Plank-Einstein Relation)}$$

This equation is used in a simple conservation of energy relation to explain the phenomena mentioned earlier: the photoelectric effect. This relation was used in this lab to extrapolate the value for Plank's constant and the work function of the photo-cathode (Φ).

$$K_{max} = hf - \Phi \qquad \text{(Photoelectric effect relation)}$$

This equation describes the maximum kinetic energy the electrons can have when they are freed from the metal by incident photons. This energy is simply the difference between the incoming photon's energy (hf), and the energy it takes to free the electron from the metal (Φ).

II. MATERIALS AND METHODS

A. Tools

In this lab a mercury discharge lamp was used as a light source. This light was shone through an optical filter that prevented all but one wavelength to pass through and come in contact with the photo-cathode. This photo-cathode was attached to a circuit containing an analog picoAmmeter in series. A digital voltmeter and controllable voltage source was attached to this circuit as well as visible in Figure 1.

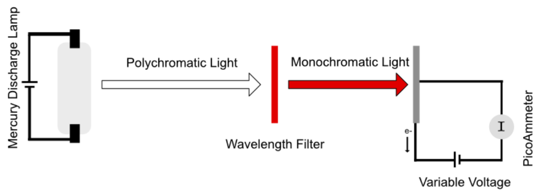

Figure 1: Display of Lab Setup

B. Procedure

1. The picoAmmeter was zeroed. After this was done caution was taken to not moving the apparatus or wires as this may effect the current. The mercury discharge lamp was turned on and aligned with the photo-cathode.

2. One of the four optical filters was attached to the photo-cathode as to block all but the allowed wavelength of light specific to the optical filter from reaching the photo-cathode.

3. The voltage, starting at zero, was increased until $\frac{dI}{dV}$ obviously approached zero, or the current began to approach the asymptote depicted in Figure 3. The voltage at which this occurred was recorded along with the wavelength permitted by the optical filter. For each filter each member of the group made one measurement. These were later averaged to reduce error in this measurement.

4. The filter was removed and steps 2-3 were repeated with each of the four optical filters.

5. After the data points for each wavelength were averaged they were plotted, a best-fit line was created, and Plank's constant along with the work function of the metal were extrapolated.

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III. RESULTS AND ANALYSIS

A. Data

Wavelength (Åm)	4047	4358	5461	5770
Average $V_{stop}(V)$	-1.82	-1.569	-0.946	-0.872

Table I: Collected V_{stop} Averages

Calculated h	Percent Error	Calculated Φ
[6.9766±0.754] x10^{-34} Js	5.29%	1.42±0.297 eV

Table II: Calculated h, percent error, Φ, and work function

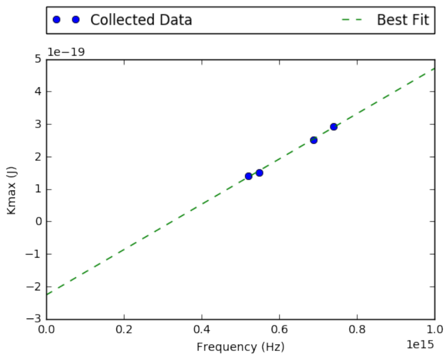

Figure 2: Collected Data with Best Fit Line

B. Calculations

The photoelectric effect relation, mentioned earlier, was used to extrapolate Plank's constant and the work function of the photo-cathode. Each wavelength/voltage pair was graphed as shown in Figure 2. The wavelengths were converted to frequencies using Equation 1

$$c = f\lambda \tag{1}$$

To convert the voltages to fit the Y axis (K_{max}), they were multiplied by the charge of an electron (-e).

$$-eV_{stop} = U = K_{max} \tag{2}$$

Assuming that the potential energy U is equivalent to K_{max} is valid because idealistically the current measured by the picoAmmeter would be zero when the potential provided by the retarding voltage was greater than or equal to the maximum kinetic energy of the freed electrons and thus preventing them from traveling through the wire. Since the V_{stop} is measured at the first point "no current" is observed, K_{max} is equivalent to U.

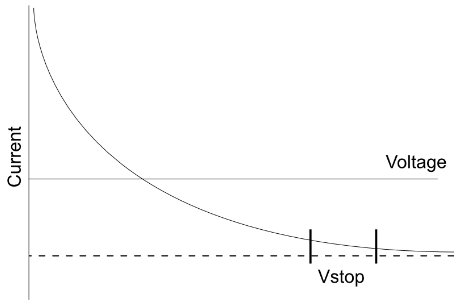

Figure 3: Relation Between Voltage and Current

Theoretically the current should zero out above a threshold voltage. However due to impurities, or crystallographic defects, in the phototube electrons are occasionally released from the photo-anode, creating a negative current called dark current. [1] This causes the current to approach an asymptote as voltage is increased as seen in Figure 3. V_{stop} is the voltage at which the current becomes relatively constant.

Once these points were generated, a best fit line was calculated using numpy.polyfit which returns the coefficient values of the best fit line in order of degree. Numpy.polyfit uses least-squares calculations to find these coefficients. Along with the best fit coefficients, numpy.polyfit can return the co-variance matrix. Taking the roots of the diagonal of this matrix returns the standard deviation of the extrapolation. The uncertainties provided in Table II have a 99.7% certainty (within 3 standard deviations). The code to generate plots and calculate data for this lab can be found here.

The uncertainties were obtained by averaging the co-variance error and the least-squares error. These account for how well the best-fit line matched the data and the accuracy of the original measurements. The least-squares uncertainty was calculated as follows.

$$\delta h = \sqrt{\sum_{i=1}^{N}(\frac{\partial h}{\partial x_i}\delta x_i)^2 + \sum_{i=1}^{N}(\frac{\partial h}{\partial y_i}\delta y_i)^2} \tag{3}$$

where

$$\frac{\partial h}{\partial x_i} = \frac{1}{N}\frac{(\langle x^2\rangle - \langle x\rangle^2)(y_i - \langle y\rangle) - (\langle xy\rangle - \langle x\rangle\langle y\rangle)(2x_i - 2\langle x\rangle)}{(\langle x^2\rangle - \langle x^2\rangle^2}^2 \tag{4}$$

and

$$\frac{\partial h}{\partial y_i} = \frac{1}{N}\frac{x_i - \langle x\rangle}{\langle x^2\rangle - \langle x\rangle^2} \tag{5}$$

- N is the number of points, four in this case
- $\langle x\rangle$, $\langle y\rangle$ is the mean value of x and y respectively
- $\langle x^2\rangle$, $\langle y^2\rangle$ is the mean value of the squares of x and y respectively

- $\langle xy \rangle$ is the average of x*y
- x_i, y_i is the current value of x and y respectively.

The uncertainty in the y-intercept was calculated in a similar fashion which can be read about at http://www.learningmeasure.com. An uncertainty of ±10nm was used for the wavelength filters based on data found for similar models online.

C. Summary

As visible in Table II, the value for Plank's constant (h) was accurately calculated from the collected data. The work function for the photo-cathode however was calculated to be unrealistically low.

Based on the information collected and calculated, the hypothesized photo-cathode metal is likely cesium. Cesium alone has a work function of 2.1 eV [2] In a previous study, a photo-cathode of cesium in combination with Antimony (Sb) had a photoelectric work function of 1.5 depending on the method of measurement which is within the margin of error for the calculated Φ. [3] This alloy (Cs_3Sb) is a likely contender for the unidentified photo-cathode not only because of its similar work function, but because its melting point is higher than cesium's, which is liquid at room temperature and thus likely not the photo-cathode.

The error in this lab may lie in the ambiguity surrounding the measurement of V_{stop} in step 3 of the procedure. As mentioned previously the impurities in the phototube cause stray electrons to generate a negative current. The method for which V_{stop} was determined is flexible in the sense that the point at which the current is deemed as "stationary" depends on a baseline that the experimenters set as the current is never truly unchanging. Throughout the separate trials this baseline was kept constant. This constant potential offset in measuring V_{stop} could explain the assumed inaccurate Φ measurement and accurate h measurement. Plank's constant - which when extrapolated is the slope of the line plotted in Figure 2 - would not be affected by a vertical translation of the function as this does not change the function's slope. The work function however is the y-intercept of the line graphed in Figure 2 and is effected by a vertical translation. This vertical translation could have been caused if the measured V_{stop} values were consistently above the actual values. This would increase the negative y-intercept and result in an impossibly low work function.

To remove this potential error in the future one should test the measurement of a V_{stop} and compare it to a calculated V_{stop} (using the accepted value for h). The difference between these two values could be used to correct the remaining V_{stop} measurements allowing for a more accurate determination for Φ if it was the case that V_{stop} was being measured incorrectly.

To improve the accuracy of Plank's constant more data points could be collected at a wider range of frequencies. The error in this lab may in part be due to the low number of tested frequencies. With only four points each one has a large impact in the line's slope. The nearly 10% uncertainty was due to a low number of points and would decrease inversely with the number of collected data points based on equation 3.

IV. CONCLUSION

With the goal of finding Plank's constant and measuring the work function of an unknown metal success was variable. The value for Plank's constant was measured accurately - within 5% of the currently accepted value - while the calculated work function has an unrealistically low value. In future labs the number of data points collected should be increased to limit the uncertainty and enhance the accuracy of the calculated value. V_{stop} measurement methods should be double-checked. A common alloy used for photo-cathodes was found to have a work function within the margin of error of the calculated one and thus is a likely contender for the unidentified metal.

[1] *Physics of solar cells*, http://depts.washington.edu/cmditr/modules/opv/physics_of_solar_cells.html, accessed: 2016-11-23.

[2] *Work functions for photoelectric effect*, http://hyperphysics.phy-astr.gsu.edu/hbase/Tables/photoelec.html, accessed: 2016-11-23.

[3] T. Sakata, Journal of the Physical Society of Japan **8**, 723 (1953).