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Resonant Frequencies in Closed Pipes

Introduction

This lab required the making a standing wave in a closed pipe. A standing wave is a vibrational pattern within a medium, where the frequency of the source causes reflected waves to interact with new forming waves, emanating from the source, to constructively interfere ("Standing Waves"). A node is a place where a trough and crest meet creating destructive interference and there is no amplitude. An antinode is where either two troughs or two crests meet creating constructive interference and the amplitude is doubled. When talking about sound waves, amplitude has a direct correlation to volume. At the antinodes, the volume is at its maximum and at the nodes there is no sound.

Sound cannot travel without a medium. Sound is a mechanical wave meaning it needs to have a medium to be able to project. A medium is a material which energy can pass through. Sound waves are able to travel due to particle interaction. A wave, by definition, is energy moving through a medium. When sound waves travel, they transfer their energy to a particle which then causes it to become displaced and bump into other particles and transfer its energy and so on. The speed of sound all depends on the type of wave and medium. For this lab it would make sense that the speed of sound is constant because the different tuning forks produce the same types of waves and the sound waves they produce will travel through the same medium. If the medium was changed, then the speed of sound would not be constant. For example, sound travels four times faster in water than air because water particles are more compacted together than air molecules.

It is no coincidence that at a certain pipe length, the volume of the sound the tuning fork produces gets significantly louder. The sound gets louder when the pipe is at a certain length is because an antinode forms at the opening. This makes sense logically because it takes a fourth of a wave for a trough to travel to the barrier in a closed pipe before it inverts to a crest and travels a fourth of a wave back to its starting point where it constructively interferes with another crest. The frequency of the tuning fork is already predetermined by the tuning fork, so the issue is getting the length of the tube to correctly correspond with the wavelength. As explained before the velocities have to be the same due to the same medium being used, so that means the frequency and wavelength are inversely related. This is due to the equation $v = f\lambda$. The equation for the wavelength is $\lambda = 4(L + 0.3d)$ where L is length and d is diameter. Diameter is constant, so that means length is the only changeable variable in the equation and that means the wavelength can be manipulated to create the fundamental, where an antinode is at the opening.

The objective of the lab was to figure out the speed of sound in air. The speed of sound was found using two different formulas. One of the formulas was $v = 331 + 0.6T$ where T is temperature in Celsius. The other formula was $v = f\lambda$ where f is frequency and λ is wavelength. The frequency was given with the tuning forks and wavelength was found using the formula mentioned in the last paragraph. The hypothesis was that if the medium stayed constant then there should be no difference between the two velocities.

Results

Diameter of Tube = 0.038m Actual Speed (based on room temperature) = 343.6 m/s

Table 1.1: Collected and Calculated Data

Trial	**Frequency (Hz)**	**Tube Length (m)**	**Wavelength (m)**	**Experimental Speed (m/s)**	**% Error (based on speed of air)**
1	293	0.278	1.16	340.0	1.05
2	329	0.25	1.04	344.6	0.29
3	384	0.217	0.9136	350.8	2.1
4	440	0.189	0.8016	352.7	2.65
5	523	0.15	0.6456	337.8	1.69

The frequency trials were not linearly spaced. The 293 Hz trial was closer to the 329 trail and the 523 Hz trial was farther removed from the rest of the trials. The Tube Length and Wavelength data follow a descending pattern as the frequency rises, suggesting an inverse relationship. The Experimental Speed rises with the frequency with the exception of 525 Hz trial, where the Experimental Speed drops below the speed for 293 Hz trial. Percent error was relatively small for this lab. The graphics on the following page are spectrograms, each representing a spectrum of frequencies over time. The spectrograms were created using recordings of trials 1,4 and 5. The frequency spectrum was decompressed from the recordings using the Fast Fourier Transform (FFT) algorithm along with the amplitudes, then plotted against time. The resonant frequencies from each recording can be seen in the dark red horizontal lines present in each graphic. The amplitude scale (supposed to be in dB) is inaccurate, but is still scaled proportionally. If the dB measurements were accurate, we would all be in trouble, but high school physics labs usually don’t involve things with pressure waves on the scale of 30,000 tons of TNT (unfortunately). In all three spectrograms there is a lot of low frequency noise.

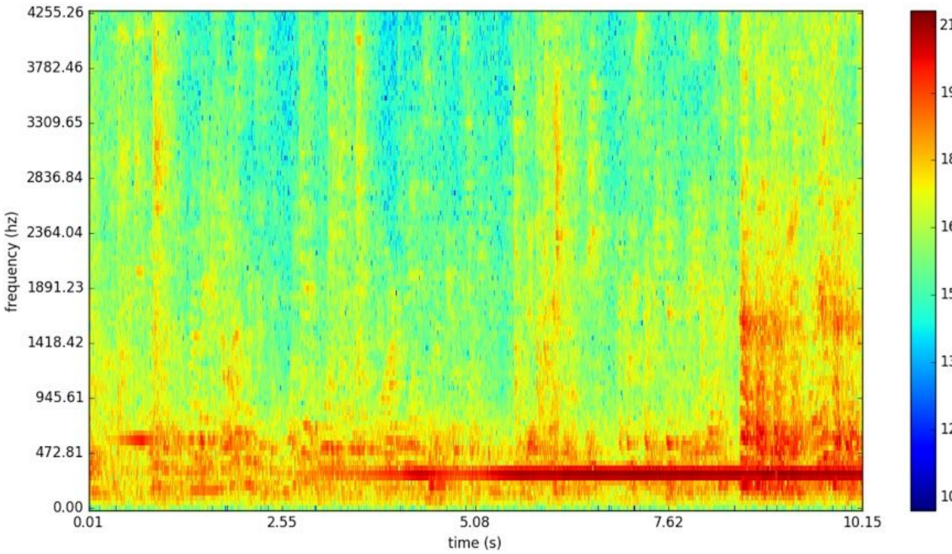

Figure 1.1: Spectrogram of 293Hz trial (a in dB)

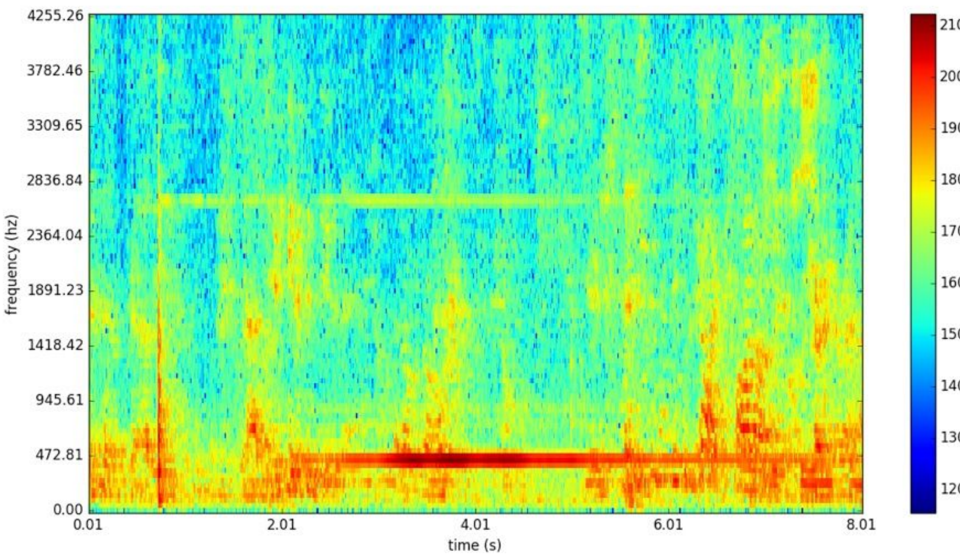

Figure 1.2: Spectrogram of 440Hz trial (a in dB)

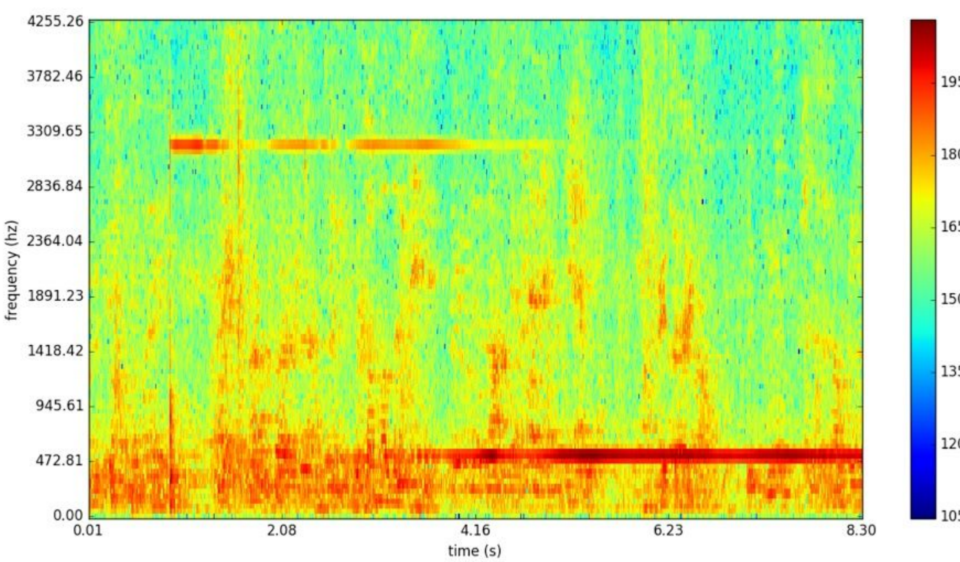

Figure 1.3: Spectrogram of 523Hz trial (a in dB)

Discussion

Taking a closer look at Figure 1.2-1.3, the time the tuning fork was initially struck can be seen as a spike in amplitude across the frequency spectrum (around 0.6 seconds for both). Most of the frequencies quickly diminish back to normal levels, with the exception of one frequency in each graph. At this point the tuning fork is being moved above the pipe, and the pipe is being moved to find the correct length. In Figure 1.1 there is no obvious striking of the tuning fork, and no higher-pitched frequency to go along with it. In Figure 1.2, a frequency of approximately 2700 Hz stays active throughout most of the recording, dying off towards the end. In Figure 1.3 the generated frequency is higher pitched, around 3200 Hz. In figure 1.3 the higher frequency seems to die off as the fundamental frequency gains amplitude. In both of the recordings for the 440 Hz and 523 Hz trials, this higher pitch can be clearly heard as soon as the tuning fork is initially struck. Neither of these frequencies match any overtones the tuning fork could generate, as neither is a multiple of the fundamental. Dividing the generated high frequencies by the fundamentals of each tuning fork results in a similar ratio for each, around 6.15. Based on research done by a graduate from Pennsylvania State University featured in an article titled, "Vibrational Modes of a Tuning Fork", tuning forks vibrate in different ways, resulting in different effects. "Fundamental Mode" is the standard mode, in which the frequency printed on the tuning fork is generated. The two tines of the fork more together and apart symmetrically. "Clang Mode" is similar, except the tines of the fork touch with every oscillation. According to the article, this "mode" generates a frequency "roughly 6.26 times higher than the fundamental", matching our earlier ratio. This also explains the absence of a higher frequency in Figure 1.1. Striking methods varied throughout the trials, from hitting the tuning fork on the sole of a shoe

(Trial 1-3) to tapping it on a solid surface (Trial 4-5). It is likely that hitting the tuning fork on the sole of a shoe didn't cause the tuning fork to enter "Clang Mode", which generated the higher frequencies. The article describes many other modes of tuning fork vibration, but none of them are likely to be present in this lab.

Based on the amplitude of the higher frequencies in Figure 1.1 and Figure 1.2 it is likely that the tuning fork in trial 5 was struck harder, because the "clang" frequency has a higher amplitude. With further experimentation the force a tuning fork was struck by could be found by analyzing the frequencies it generates afterword by accounting for the "Clang Mode". Based on the data, the decay of higher "clang" frequencies is likely exponential. Figure 1.3 has a much higher amplitude to start with, but the amplitude quickly drops compared to the decay of the amplitude of the higher frequency in Figure 1.2. This was likely due to to the physical collisions occurring during the "Clang Mode" of the tuning fork, which may remove more energy from the system proportional to how violent the collision between the prongs was. Further experimentation focusing in this area would allow less speculation on the subject.

The speed of sound in air should not change or vary as long as the temperature and air conditions are kept constant. This is shown by the formula $v = 331 + 0.6T$, as discussed in the introduction. Since the temperature stayed constant throughout the course of this experiment, the speed of sound from this formula never changed. The formula $v = f\lambda$ also gives the speed of sound in air, which means every calculation should come out to be the same between the two equations, no matter the frequency or varying wavelengths. This was not the case, as shown by the percent errors in Table 1.1. Every single calculation with the temperature formula came out to be 343.6 m/s. The largest percent error was only 2.6% with the tuning fork of a 440 Hz frequency. The error should have been zero. The only factors capable of changing the data, according to the equations, are the wavelength and frequency. The frequencies were never measured, simply marked and copied, and any alteration to the tuning forks in the years of use would change their frequencies, adjusting the data.

The wavelength could have been an altered measurement in many ways. The formula used for wavelength was $\lambda = 4(L + 0.3d)$. This relies on the length of the tube and on the diameter. The diameter may not have been an exact measurement, but the same one was used for each calculation, and therefore if it was the only source of slight error then the error for each trial would have been exactly the same, which was not the case. Thus the error is left in the length of the tube. In a closed pipe, the length of the first harmonic is a quarter of the wavelength. This means if each of the recorded lengths is multiplied by four, the result should be equal to the wavelength that was gathered from the above formula. All of them, however, are shorter than the recorded wavelengths after being multiplied. For example, the tube length of .217m multiplies to a .868 m wavelength, where the recorded is .914 m. However, multiplying .868 by the frequency of, in this case, 384 Hz, comes out as 333.3 m/s, which is farther from 344.6 m/s than the already recorded 350.8 m/s. The smaller number is logical as it is a smaller wavelength, and the lower accuracy is reasonable as well, as $L=\lambda/4$ does not specifically include the diameter of the tube.

Water was used to close off one side of the pipe for the sound waves to reflect off of, yet energy can travel through water. This grants reason to believe the water absorbed some of the energy of the wave as it bounced off. If the water was absorbing energy, that would mean the collision was inelastic and that the water was a non-rigid boundary. This does not make sense, however, as waves do not invert off of non-rigid boundaries. The waves in this lab would have had to invert as it was a closed pipe. By definition of a closed pipe, the water had to be a rigid boundary. Therefore the boundary was neither rigid nor soft, it was in between. Some of the waves were reflected from the boundary and some were transmitted across. This is an example of a wave reflecting from an impedance discontinuity. The amplitude of each of these parts of the wave would be different as they would be in different mediums. The one in the original medium would be calculated by $\xi_r=(Z_1-Z_2)/(Z_1+Z_2)\xi_1$, where the impedance, Z, of an object is given by multiplying the mass density and wave speed. The part of the wave in the second medium would be calculated with $\xi_2=(2Z_1)/(Z_1+Z_2)\xi_1$. ("Reflection of Waves from Boundaries").

Lastly, wavelengths may have differed due to improper readings or the human error when holding the tube. If not held straight, the sound would travel down the tube at an angle and therefore, as waves do, reflect off the boundary in the opposite direction from which it came, similar to the way radio telescopes reflect sound waves to their focus points. In the tube, the wave would crash directly into the wall of the tube in front of it, and thereafter reflect off of that, and bounce back and forth until it went out of the tube. This most likely did not happen, as the error would be substantially larger than it was with the number of reflections that would be occurring in that scenario. It would have been very challenging to find the loudest point. The waves were loudest at the antinodes, because that is constructive interference as opposed to the destructive at a node. Constructive interference is a crest meeting a crest or a trough meeting a trough and is therefore the addition of two added together, causing double the amplitude and the loudest point of the wave. Knowing what interferes with waves, what can cause error, where waves are loudest, and the optimal length of a tube can help the military develop non-lethal weapons to safely disperse crowds. To further this lab it would be completed with a boundary known as rigid or non-rigid, or the impedance for a boundary in between would be calculated and accounted for.

References

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An Interactive Guide To The Fourier Transform. (n.d.). Retrieved February 10, 2016, from http://betterexplained.com/articles/an-interactive-guide-to-the-fourier-transform/

"Formation of Standing Waves." *Formation of Standing Waves*. N.p., n.d. Web. 10 Feb. 2016.

Footnote:

A great interactive guide to understanding the Fast Fourier Transform (FFT) can be found at: http://betterexplained.com/articles/an-interactive-guide-to-the-fourier-transform/
The FFT equation has an uncounted number of applications, some of which include file compression, sound manipulation (bass boosting etc), and anti-earthquake architectural design.

The code used to generate the spectrograms was based on code that can be found at: http://www.frank-zalkow.de/en/code-snippets/create-audio-spectrograms-with-python.html

Basic sound unpacking and sampling was learned from: http://samcarcagno.altervista.org/blog/basic-sound-processing-python/

Decibel equivalents were found at:http://www.decibelcar.com/articles/40-everything-else/87-dbequivalent.html

Vibration Modes of a Tuning fork is a well written article with helpful graphics that goes in-depth on the different behaviours of tuning forks. http://www.acs.psu.edu/drussell/Demos/TuningFork/fork-modes.html